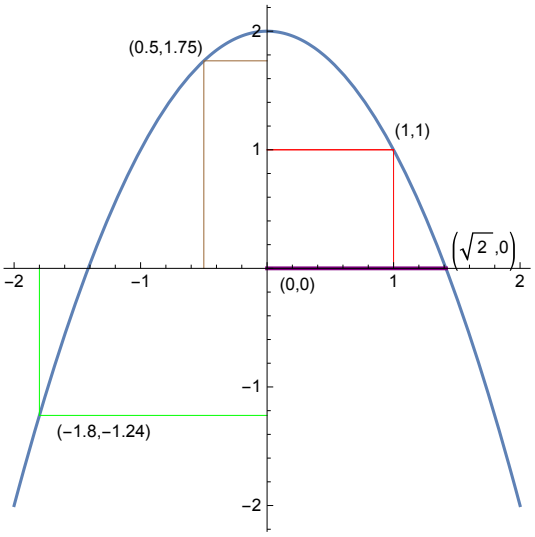
- Continuous functions of one variable

ContFunc

```
2 - (1.8)^2
```

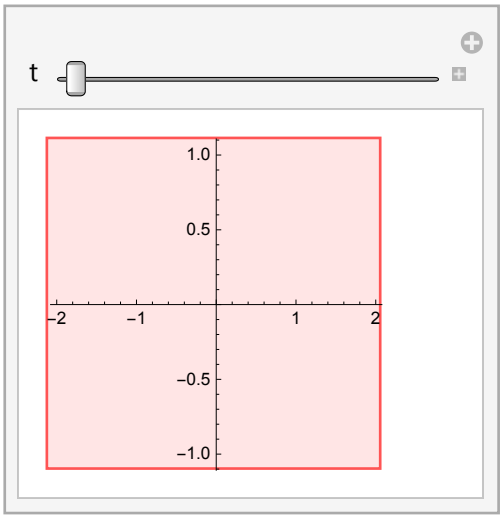
```
-1.24
```

```
Show[Plot[2 - x^2, {x, -2, 2}],
 Graphics[{Red, Line[{{0, 1}, {1, 1}, {1, 0}}], Line[{{0, 1}, {1, 1}}],
   Brown, Line[{{-.5, 0}, {-.5, 1.75}, {0, 1.75}}], Green,
   Line[{{-1.8, 0}, {-1.8, 2 - (1.8)^2}, {0, 2 - (1.8)^2}}],
   Thick, Purple, Line[{{Sqrt[2], 0}, {0, 0}}],
   Black, Inset["(1,1)", {1 + .15, 1 + .15}],
   Inset["(0.5,1.75)", {-.5 + -.3, 1.75 + .1}],
   Inset["(-1.8,-1.24)", {2 - (1.8)^2 - .05, 2 - (1.8)^2 - .15}],
   Inset["(√2 ,0)", {Sqrt[2] + .3, .15}], Inset["(0,0)", {.24, -.15}]}
 ], AspectRatio → 1, ImageSize → 275
]
```




```
uppar[t_] := 2 - t^2
```

```
Manipulate[
 Show[
  Plot[uppar[x], {x, -2, 2}],
  Graphics[{Red, Line[{{t, 0}, {t, uppar[t]}, {0, uppar[t]}}]}
  ],
  AspectRatio → 1, ImageSize → 175
 ], {t, -2, 2}]
```




```
upparmovie = Table[Show[
     Plot[uppar[x], {x, -2, 2}],
     Graphics[{Red, Line[{{t, 0}, {t, uppar[t]}, {0, uppar[t]}}]}
     ],
     AspectRatio → 1, ImageSize → 175
    ], {t, -2, 2, .1}];
```

```
Export[
 "Z:\www\MM\Mathematica\Function Representations\upparmovie.gif", upparmovie]
```

Export::nodir : Directory Z:\www\MM\Mathematica\Function Representations\ does not exist. >>

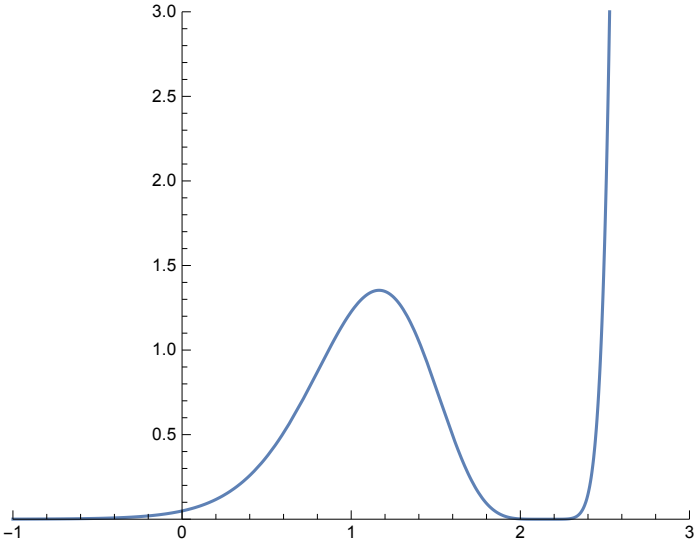
Export::noopen : Cannot open Z:\www\MM\Mathematica\Function Representations\upparmovie.gif. >>

```
$Failed
```

ContFunc2

```
foolyou[x_] := .0002 ((x^3 - 10) / (3 Exp[-x] + 1))^6
```

```
Plot[foolyou[x], {x, -1, 3}, PlotRange → {{-1, 3}, {0, 3}}, AspectRatio → 3 / 4]
```




```
10^(1/3) // N
```

```
2.15443
```

```
Solve[x^3 - 10 == 0, x]
```

Solve::ivar : 0.9999418638594934` is not a valid variable. >>

```
Solve[False, 0.999942]
```

```
-(-10)^(1/3) // N
```

```
-1.07722 - 1.8658 i
```

```
(-1)^(2/3) 10^(1/3) // N
```

```
-1.07722 + 1.8658 i
```

```
foolyou'[x]
```

```
1.40809
```

```
foolyou''[x]
```

```
-5.94097
```

```
NSolve[foolyou'[x] == 0, x, Reals]
```

NSolve::ivar : 0.9999418638594934` is not a valid variable. >>

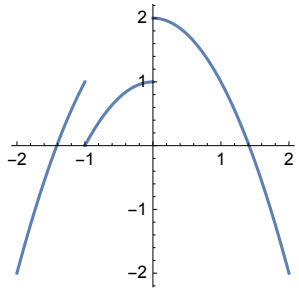
```
NSolve[False, 0.999942, Reals]
```

```
foolyou''[1.1648251336938016`]
```

```
-10.6664
```

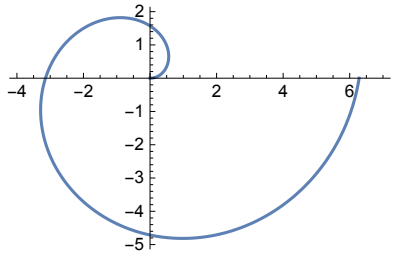
Discontinuous function

```
Remove[dfunc]

dfunc[x_] := 2 - x^2 /; x > 0; dfunc[x_] := 2 - x^2 /; x < -1;
dfunc[x_] := 1 - x^2 /; x > -1 && x < 0;

Plot[dfunc[x], {x, -2, 2}, AspectRatio → 1,
 Exclusions → {x = 0, x = -1 }, ImageSize → 150]
```



Polar Plot

```
PolarPlot[t, {t, 0, 2 Pi}, ImageSize → 200,
 PlotRange → {{-4, 7}, {-5, 2}}, AspectRatio → 7/11]
```




```
ToPolarCoordinates[{t, 1}]
```

$\{\sqrt{1+t^2}, \text{ArcTan}[t, 1]\}$

```
FromPolarCoordinates[{√(1 + t^2) , ArcTan[t, 1]}]
```

```
{t, 1}
```

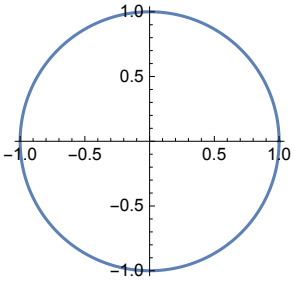
```
ToPolarCoordinates[{x, y}]
```

```
{1.00023, -3.12003}
```

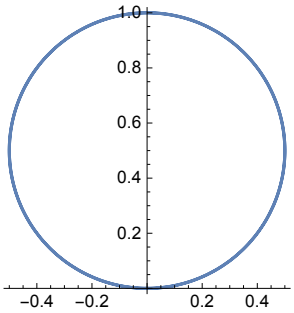
```
FromPolarCoordinates[{r, t}]
```

```
{r Cos[t], r Sin[t]}
```

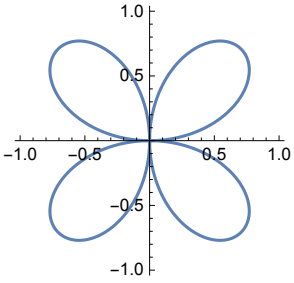
```
PolarPlot[1, {t, 0, 2 Pi}, ImageSize → 150]
```



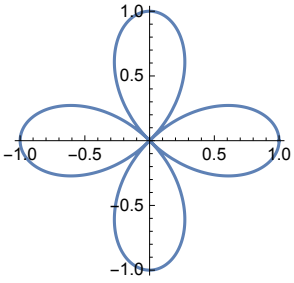

```
PolarPlot[Sin[t], {t, 0, 2 Pi}, ImageSize → 150]
```



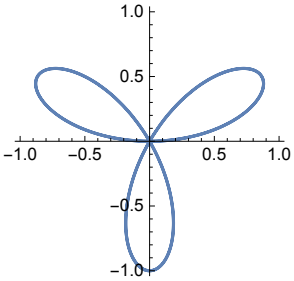

```
PolarPlot[Sin[2 t], {t, 0, 2 Pi}, PlotRange → {{-1, 1}, {-1, 1}}, ImageSize → 150]
```




```
PolarPlot[Cos[2 t], {t, 0, 2 Pi}, ImageSize → 150]
```



```
PolarPlot[Sin[3 t], {t, 0, 2 Pi}, PlotRange → {{-1, 1}, {-1, 1}}, ImageSize → 150]
```

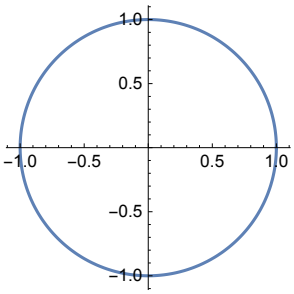


- Functions from R to R x R

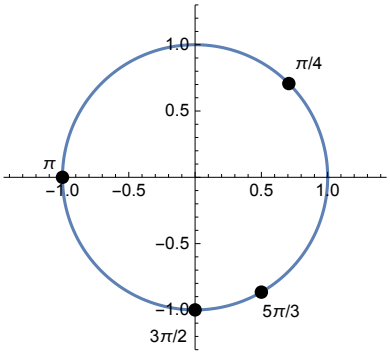
Unit Circle

Bare unit circle

```
ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 Pi}, ImageSize → 150]
```



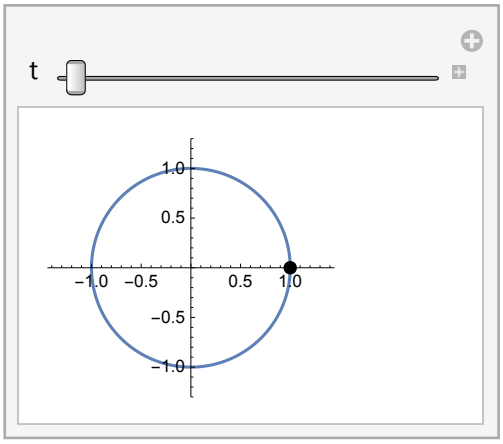
```
Show[{ParametricPlot[{Cos[t], Sin[t]},
    {t, 0, 2 Pi}, PlotRange → {-1.3, 1.3}, ImageSize → 200],
  Graphics[{PointSize[Large], Black, Point[{Cos[5 Pi / 3], Sin[5 Pi / 3]}],
    Text["5π/3", {Cos[5 Pi / 3] + .15, Sin[5 Pi / 3] - .15}],
    Point[{Cos[-Pi / 2], Sin[-Pi / 2]}],
    Text["3π/2", {Cos[3 Pi / 2] - .2, Sin[3 Pi / 2] - .2}], Point[{Cos[Pi], Sin[Pi]}],
    Text["π", {Cos[Pi] - .1, Sin[Pi] + .1}], Point[{Cos[Pi / 4], Sin[Pi / 4]}],
    Text["π/4", {Cos[Pi / 4] + .15, Sin[Pi / 4] + .15}]}]}]
```



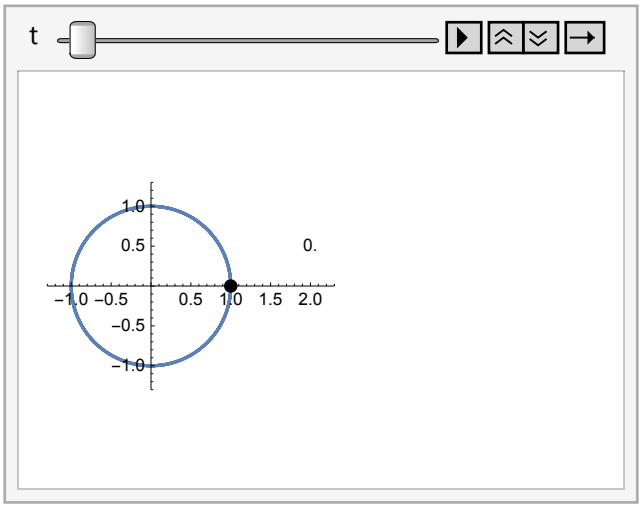

```
Sqrt[3] / 2 // N
```

```
0.866025
```

```
Manipulate[
 Show[ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 Pi}, PlotRange → {-1.3, 1.3},
   ImageSize → 200], Graphics[{PointSize[Large], Black, Point[{Cos[t], Sin[t]}]}],
  ImageSize → 150], {{t, 0}, 0, 10 Pi}]
```



```
Animate[
 Show[
  ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 8 Pi},
   PlotRange → {{-1.3, 2.3}, {-1.3, 1.3}}, ImageSize → 200],
  Graphics[{PointSize[Large], Black, Point[{Cos[t], Sin[t]}], Text[t, {2, .5}]}],
  ImageSize → 150], {{t, 0}, 0, 10 Pi}, AnimationRunning → False]
```




```
outstring[t_] := StringJoin["t=", ToString[NumberForm[N[(t / Pi)], {3, 3}]], "π"]
```

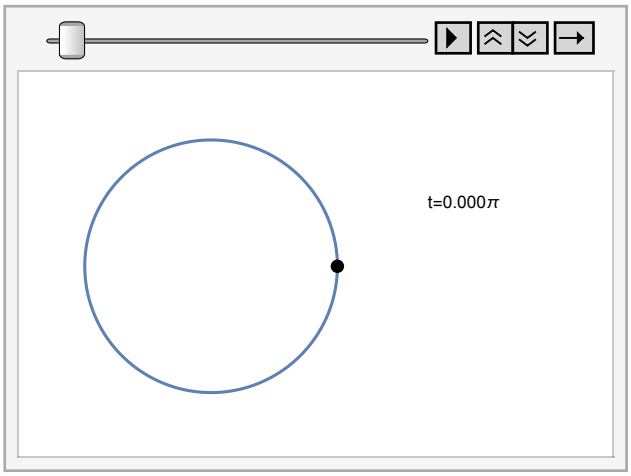
```
outstring[1.3]
```

```
t=0.414π
```

```
Table[Show[
    ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 Pi},
     PlotRange → {{-1.3, 2.5}, {-1.3, 1.3}}, ImageSize → 200],
    Graphics[{PointSize[Large], Black, Point[{Cos[t], Sin[t]}],
      Text[NumberForm["t=" N[(t / Pi)], {3, 3}] "π", {2, .5}]}],
    ImageSize → 250, Axes → False], {t, 0, 2 Pi, Pi / 30}];
```

```
circlemovie = Table[Show[
     ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 Pi},
      PlotRange → {{-1.3, 2.5}, {-1.3, 1.3}}, ImageSize → 200],
     Graphics[{PointSize[Large], Black, Point[{Cos[t], Sin[t]}], Text[
        outstring[t], {2, .5}]}], ImageSize → 250, Axes → False], {t, 0, 2 Pi, Pi / 30}];
```

```
ListAnimate[circlemovie, AnimationRunTime → 1, AnimationRunning → False]
```

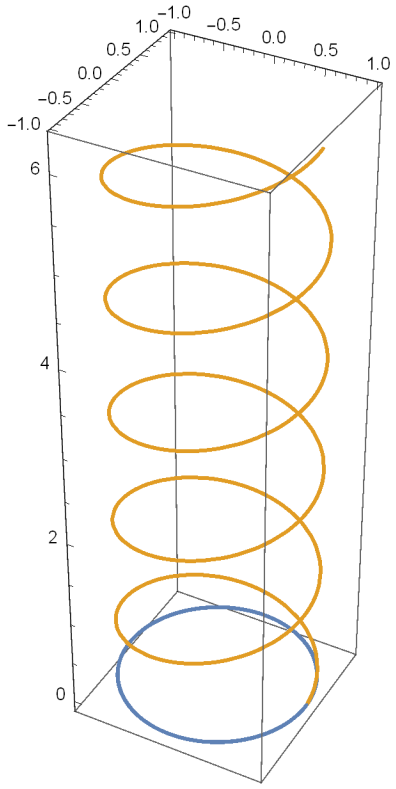



```
Export[
 "W:\www\MM\Mathematica\Function Representations\circlemovie.gif", circlemovie]
```

```
W:\www\MM\Mathematica\Function Representations\circlemovie.gif
```

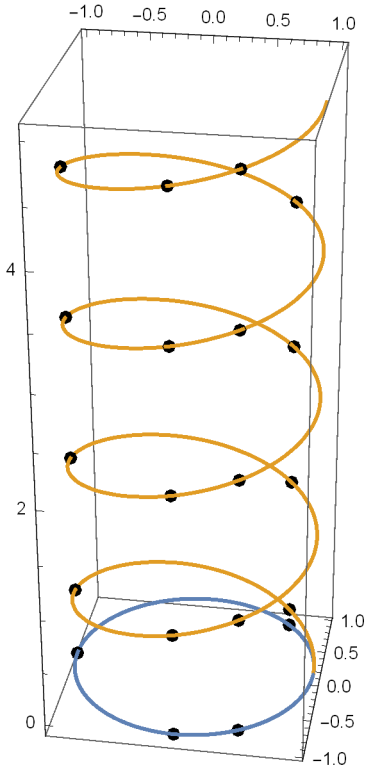
Unit circle with universal cover

```
ParametricPlot3D[{{Cos[t], Sin[t], 0}, {Cos[t], Sin[t], .2 t}},
 {t, 0, 10 Pi}, ImageSize → 200]
```

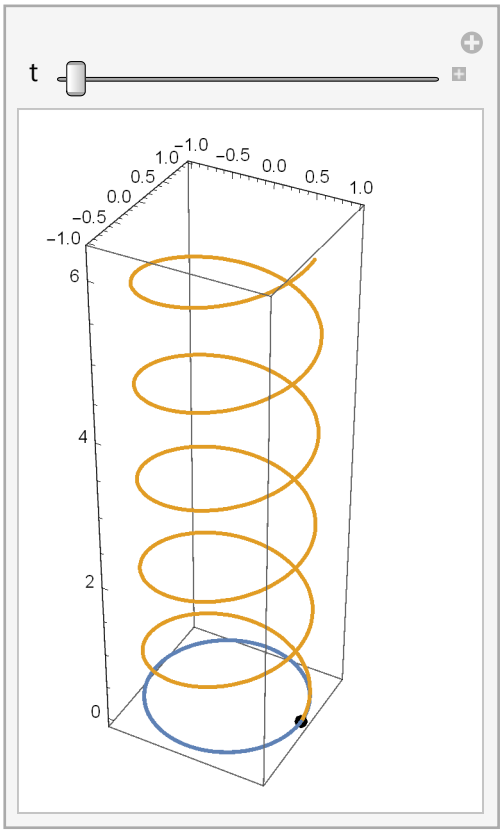


Show fibers

```
Show[ParametricPlot3D[{{Cos[t], Sin[t], 0}, {Cos[t], Sin[t], .2 t}}, {t, 0, 8 Pi}],
 Graphics3D[{PointSize[Large], Black, Point[{Cos[5 Pi / 3], Sin[5 Pi / 3], 0}],
   Point[{Cos[3 Pi / 2], Sin[3 Pi / 2], 0}], Point[{Cos[Pi], Sin[Pi], 0}], Point[
    {Cos[Pi / 4], Sin[Pi / 4], 0}], Point[{Cos[5 Pi / 3], Sin[5 Pi / 3], .2 (5 Pi / 3)}],
   Point[{Cos[3 Pi / 2], Sin[3 Pi / 2], .2 (3 Pi / 2)}], Point[{Cos[Pi], Sin[Pi], .2 Pi}],
   Point[{Cos[Pi / 4], Sin[Pi / 4], .2 (Pi / 4)}],
   Point[{Cos[5 Pi / 3], Sin[5 Pi / 3], .2 (5 Pi / 3 + 2 Pi)}],
   Point[{Cos[3 Pi / 2], Sin[3 Pi / 2], .2 (3 Pi / 2 + 2 Pi)}],
   Point[{Cos[Pi], Sin[Pi], .2 (Pi + 2 Pi)}],
   Point[{Cos[Pi / 4], Sin[Pi / 4], .2 (Pi / 4 + 2 Pi)}],
   Point[{Cos[5 Pi / 3], Sin[5 Pi / 3], .2 (5 Pi / 3 + 4 Pi)}],
   Point[{Cos[3 Pi / 2], Sin[3 Pi / 2], .2 (3 Pi / 2 + 4 Pi)}],
   Point[{Cos[Pi], Sin[Pi], .2 (Pi + 4 Pi)}],
   Point[{Cos[Pi / 4], Sin[Pi / 4], .2 (Pi / 4 + 4 Pi)}],
   Point[{Cos[5 Pi / 3], Sin[5 Pi / 3], .2 (5 Pi / 3 + 6 Pi)}],
   Point[{Cos[3 Pi / 2], Sin[3 Pi / 2], .2 (3 Pi / 2 + 6 Pi)}],
   Point[{Cos[Pi], Sin[Pi], .2 (Pi + 6 Pi)}],
   Point[{Cos[Pi / 4], Sin[Pi / 4], .2 (Pi / 4 + 6 Pi)}]
  }], Boxed → True
]
```

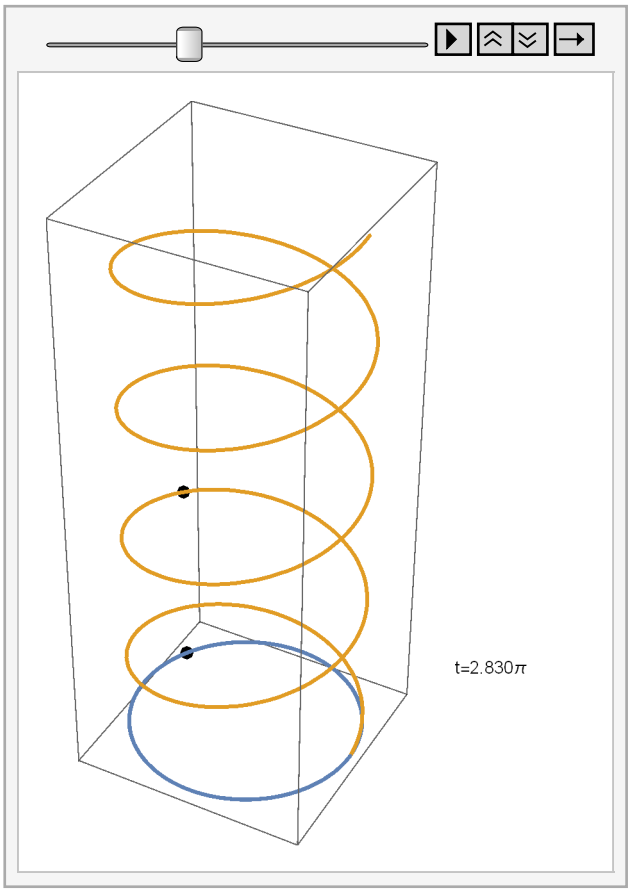


```
Manipulate[Show[ParametricPlot3D[{{Cos[t], Sin[t], 0}, {Cos[t], Sin[t], .2 t}},
    {t, 0, 10 Pi}, ImageSize → 170], Graphics3D[{PointSize[Large],
     Black, Point[{Cos[t], Sin[t], 0}], Point[{Cos[t], Sin[t], .2 t}]
    }]
 ], {t, 0, 10 Pi}]
```




```
covermovie = Table[Show[
     ParametricPlot3D[{{Cos[t], Sin[t], 0}, {Cos[t], Sin[t], .2 t}}, {t, 0, 8 Pi},
      Axes → False], Graphics3D[{PointSize[Large], Black, Point[{Cos[t], Sin[t], 0}],
       Point[{Cos[t], Sin[t], .2 t}], Text[outstring[t], {2, .5, 1}]}],
     ImageSize → 250], {t, 0, 8 Pi, Pi / 30}];
```

```
ListAnimate[covermovie, AnimationRunTime → 1, AnimationRunning → False]
```

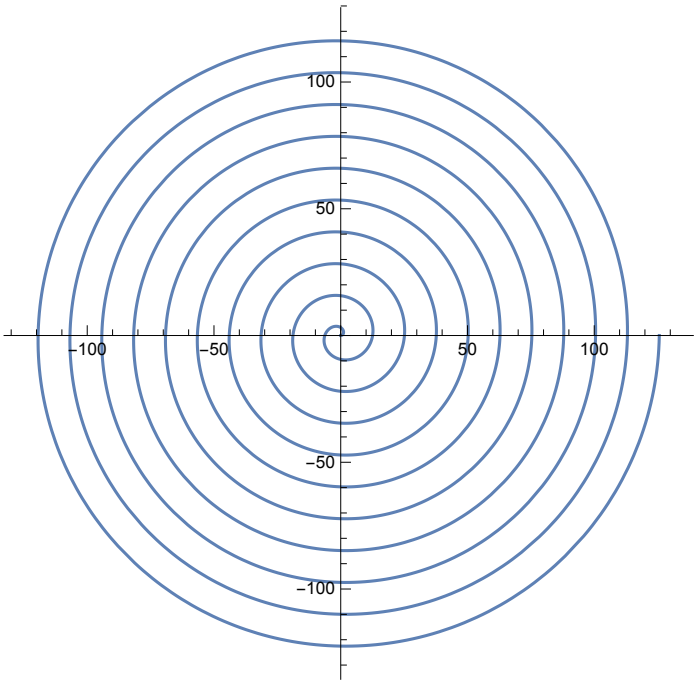



```
Export[
 "W:\www\MM\Mathematica\Function Representations\covermovie.gif", covermovie]
```

```
W:\www\MM\Mathematica\Function Representations\covermovie.gif
```

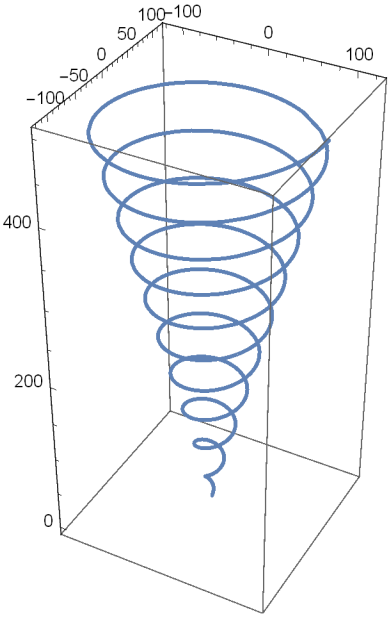
Spiral

```
ParametricPlot[{2 t Cos[t], 2 t Sin[t]}, {t, 0, 20 Pi}]
```



Spiral with universal cover

```
ParametricPlot3D[{2 t Cos[t], 2 t Sin[t], 8 t}, {t, 0, 20 Pi}, ImageSize → 200]
```



```
ParametricPlot3D[{{2 t Cos[t], 2 t Sin[t], 0}, {2 t Cos[t], 2 t Sin[t], 8 t}},
 {t, 0, 20 Pi}, ImageSize → 200]
```

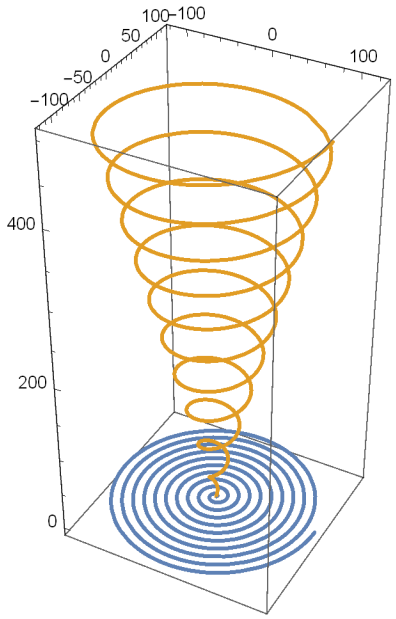
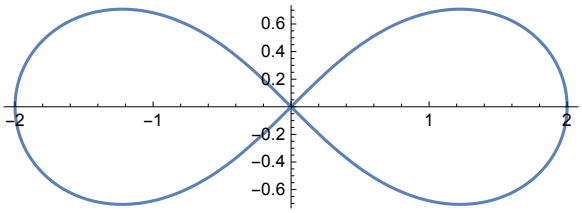


Figure eight

```
PolarPlot[Sqrt[4 Cos[2 t]], {t, 0, 2 Pi}, ImageSize → 300]
```




```
ParametricPlot[{x = Cos[t], y = Sin[2 t]}, {t, 0, 2 Pi}, ImageSize → 150]
```

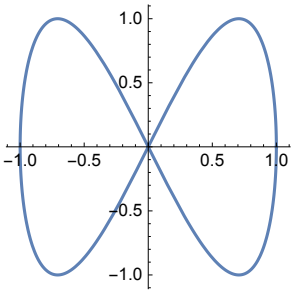
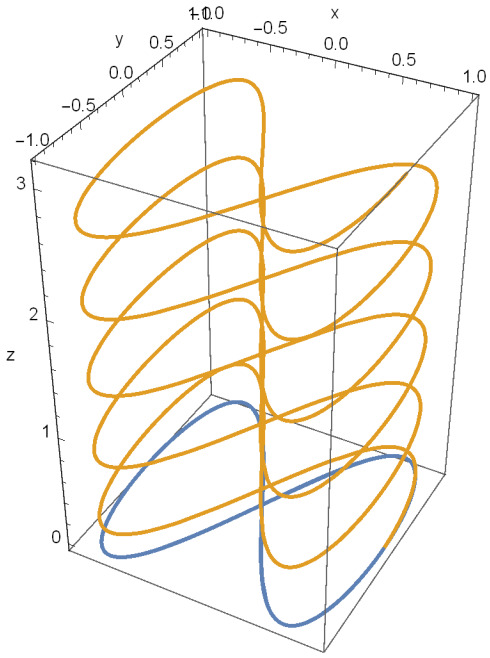
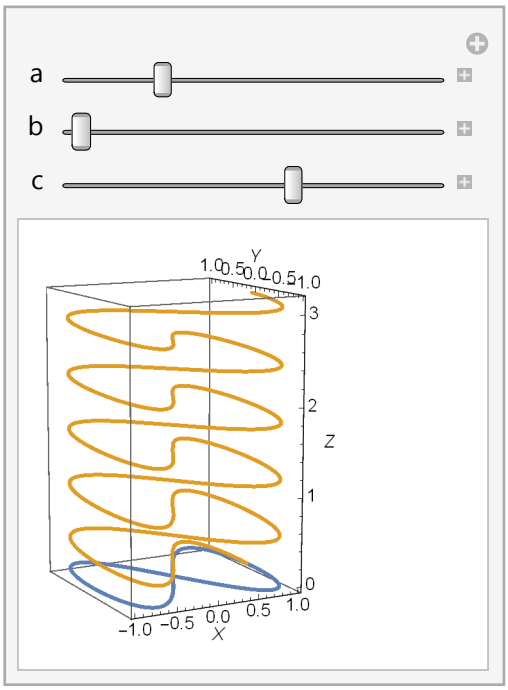


Figure 8 with universal cover

```
ParametricPlot3D[{{x = Cos[t], y = Sin[2 t], 0}, {x = Cos[t], y = Sin[2 t], .1 t}},
 {t, 0, 10 Pi}, ImageSize → 250, AxesLabel → {"x", "y", "z"}]
```



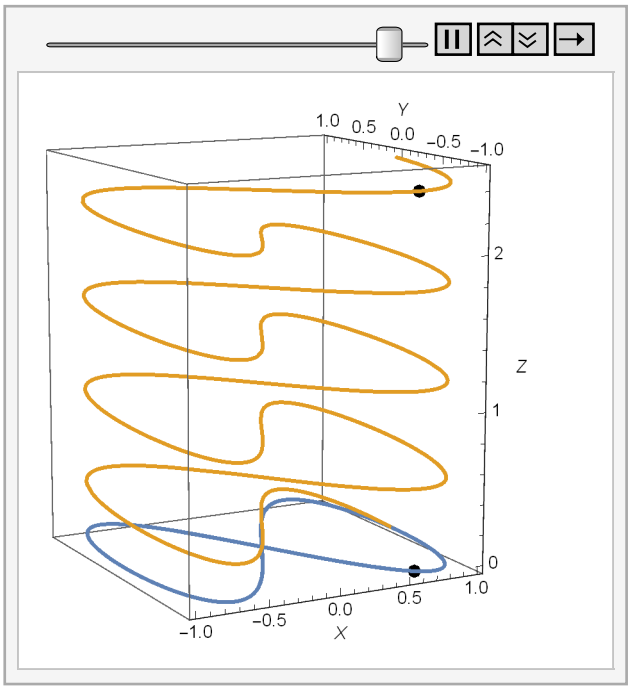
```
Manipulate[
 ParametricPlot3D[{{x = Cos[t], y = Sin[2 t], 0}, {x = Cos[t], y = Sin[2 t], .1 t}},
  {t, 0, 10 Pi}, ImageSize → 150, AxesLabel → {X, Y, Z}, ViewPoint → {a, b, c}],
 {{a, -1}, -5, 5}, {{b, 4.3}, -5, 5}, {{c, 1.92}, -5, 5}]
```




```
covermovie = Table[Show[
    ParametricPlot3D[{{Cos[t], Sin[t], 0}, {Cos[t], Sin[t], .2 t}}, {t, 0, 8 Pi},
     Axes → False], Graphics3D[{PointSize[Large], Black, Point[{Cos[t], Sin[t], 0}],
      Point[{Cos[t], Sin[t], .2 t}], Text[outstring[t], {2, .5, 1}]}],
    ImageSize → 250], {t, 0, 8 Pi, Pi / 30}];
```

```
Fig8movie = Table[Show[
    ParametricPlot3D[{{Cos[t], Sin[2 t], 0}, {Cos[t], Sin[2 t], .1 t}},
     {t, 0, 8 Pi}, AxesLabel → {X, Y, Z}, ViewPoint → {-2.65, -5, 1.12}],
    Graphics3D[{PointSize[Large], Black, Point[{Cos[t], Sin[2 t], 0}],
      Point[{Cos[t], Sin[2 t], .1 t}]}], ImageSize → 250], {t, 0, 8 Pi, Pi / 30}];
```

```
ListAnimate[Fig8movie, AnimationRunTime → 3, AnimationRunning → True]
```




```
Export[
 "W:\www\MM\Mathematica\Function Representations\covermovie.gif", covermovie]
```

```
W:\www\MM\Mathematica\Function Representations\covermovie.gif
```

```
Export["C:\Users\C&J\Desktop\Fig8movie.gif", Fig8movie]
```

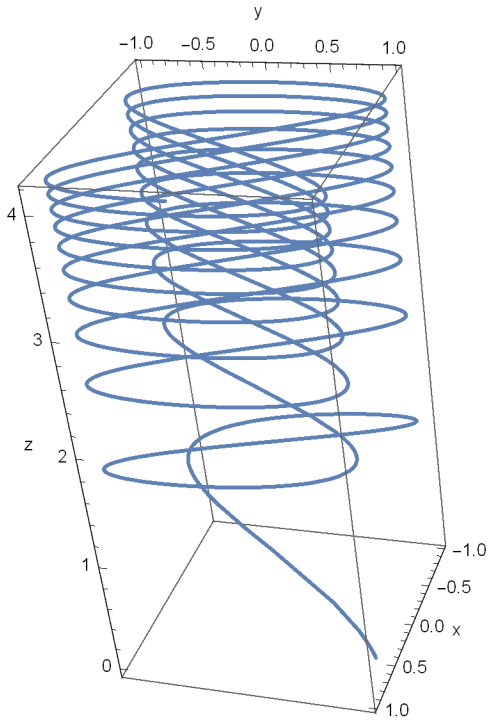
```
C:\Users\C&J\Desktop\Fig8movie.gif
```

```
Export["W:\www\MM\Mathematica\Function Representations\Fig8movie.gif", Fig8movie]
```

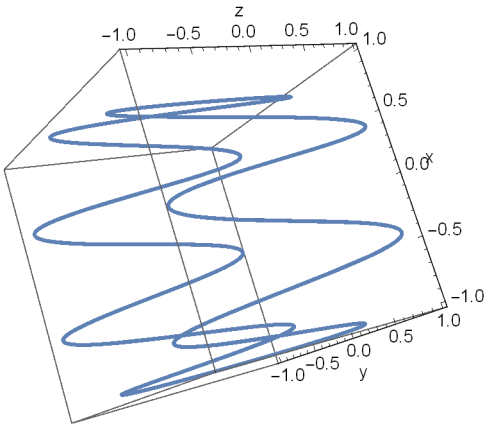
```
W:\www\MM\Mathematica\Function Representations\Fig8movie.gif
```

3 D Curves

```
ParametricPlot3D[{Cos[t], Sin[2 t], Log[t]},
 {t, 1, 20 Pi}, ImageSize → 250, AxesLabel → {"x", "y", "z"}]
```




```
ParametricPlot3D[{x = Cos[t], y = Sin[t], z = Sin[7 t]},
 {t, 0, 10 Pi}, ImageSize → 250, AxesLabel → {"x", "y", "z"}]
```




```
crownmovie = Table[Show[
    ParametricPlot3D[{{Cos[t], Sin[t], Sin[7 t]}}, {t, 0, 2 Pi}, Axes → False],
    Graphics3D[{PointSize[Large], Black, Point[{Cos[t], Sin[t], Sin[7 t]}]}],
    ImageSize → 250], {t, 0, 2 Pi, Pi / 60}];
```

```
Export[
 "W:\www\MM\Mathematica\Function Representations\crownmovie.gif", crownmovie]
```

```
W:\www\MM\Mathematica\Function Representations\crownmovie.gif
```

```
SystemOpen[DirectoryName[AbsoluteFileName[
   "W:\\www\\MM\\Mathematica\\Function Representations\\crownmovie.gif"]]]
```

Curve in space

```
shift[t_] := (-4 t^2 + 53 t) / 18
```

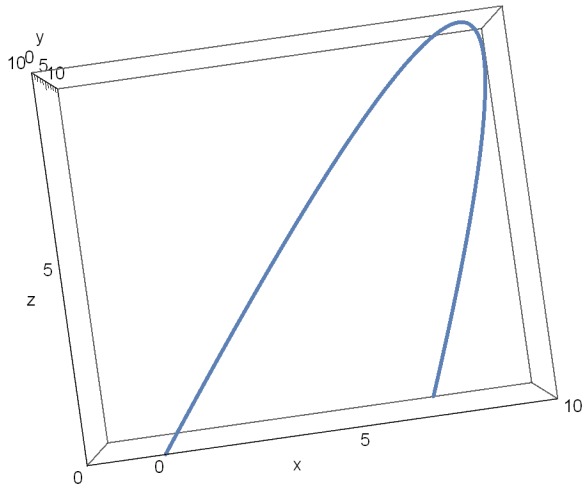
```
{shift'[t], shift''[t]}
```

$$\left\{\frac{1}{18}(53 - 8\,t), -\frac{4}{9}\right\}$$

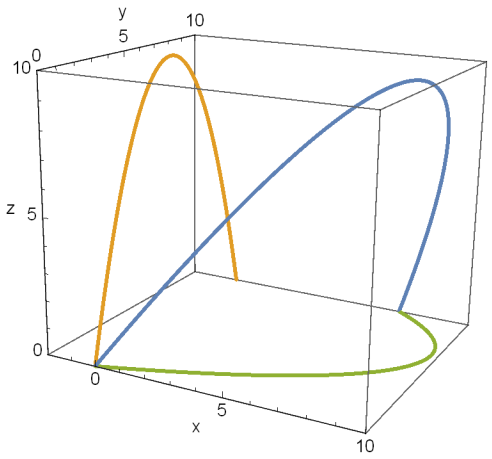
```
{shift[0], shift[10]} // N
```

```
{0., 7.22222}
```

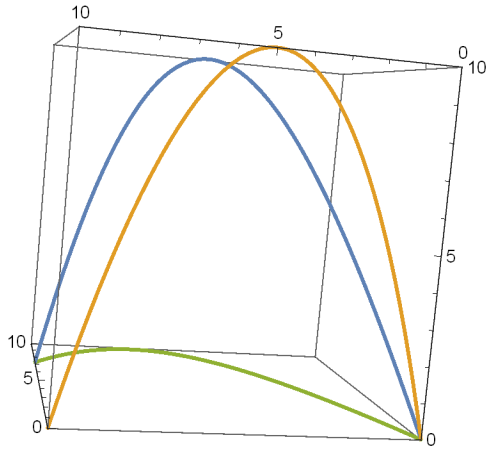
```
ParametricPlot3D[{shift[t], t, .4 (-t^2 + 10 t)},
 {t, 0, 10}, PlotRange → {{-2, 10}, {0, 10}, {0, 10}},
 AxesLabel → {"x", "y", "z"}, ImageSize → 300, ViewPoint → {0, -8, 0}]
```



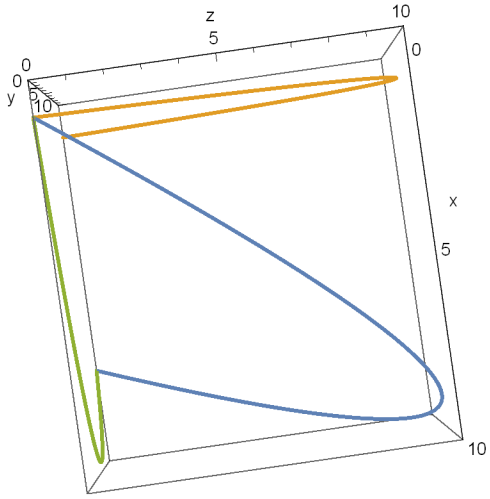
```
ParametricPlot3D[
 {{shift[t], t, .4 (-t^2 + 10 t)}, {0, t, .4 (-t^2 + 10 t)}, {shift[t], t, 0}}, {t, 0, 10},
 PlotRange → {{-2, 10}, {0, 10}, {0, 10}}, AxesLabel → {"x", "y", "z"}, ImageSize → 250]
```



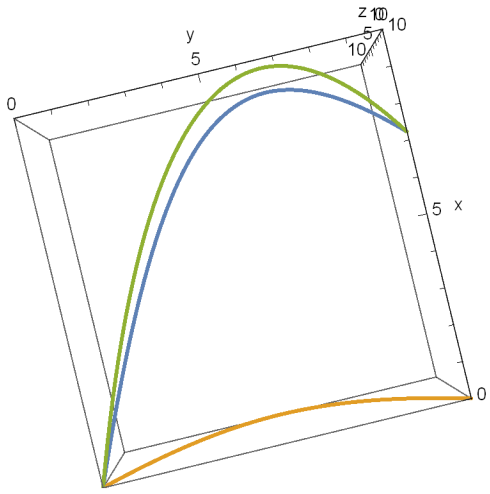

```
ParametricPlot3D[
 {{shift[t], t, .4 (-t^2 + 10 t)}, {0, t, .4 (-t^2 + 10 t)}, {shift[t], t, 0}},
 {t, 0, 10}, PlotRange → {{0, 10}, {0, 10}, {0, 10}}, ImageSize → 250]
```



```
ParametricPlot3D[
 {{shift[t], t, .4 (-t^2 + 10 t)}, {0, t, .4 (-t^2 + 10 t)}, {shift[t], t, 0}},
 {t, 0, 10}, PlotRange → {{-1, 10}, {0, 10}, {0, 10}},
 ImageSize → 250, AxesLabel → {"x", "y", "z"}, ViewPoint → {0, -6, 0}]
```

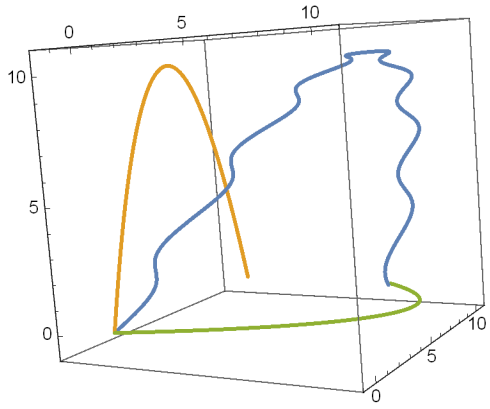



```
ParametricPlot3D[
 {{shift[t], t, .4 (-t^2 + 10 t)}, {0, t, .4 (-t^2 + 10 t)}, {shift[t], t, 0}},
 {t, 0, 10}, PlotRange → {{-0, 10}, {0, 10}, {0, 10}},
 ImageSize → 250, AxesLabel → {"x", "y", "z"}, ViewPoint → {0, 0, -6}]
```



Variation

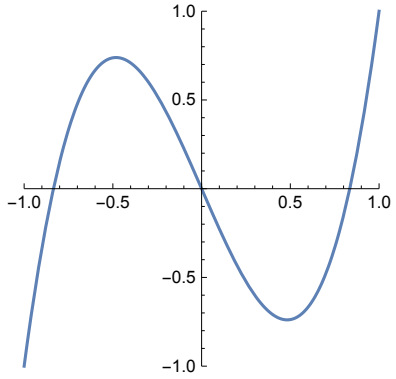
```
ParametricPlot3D[
 {{shift[t], t + Sin[5 t], .4 (-t^2 + 10 t)}, {0, t, .4 (-t^2 + 10 t)}, {shift[t], t, 0}},
 {t, 0, 10}, PlotRange → {{-2, 11}, {-1, 11}, {-1, 11}}, ImageSize → 250]
```



Experiment 2

```
exp2a[t_] := 3.3 (t^3 - t) + t
```

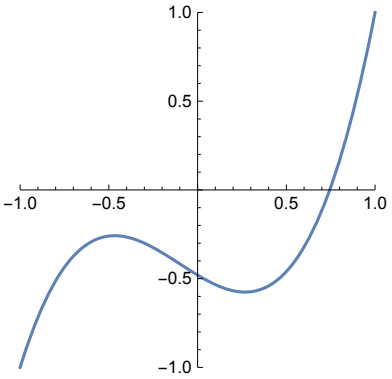
```
Plot[exp2a[t], {t, -1, 1}, ImageSize → 200,
 PlotRange → {{-1, 1}, {-1, 1}}, AspectRatio → 1]
```




```
exp2b[t_] = -(-1.6 (t - 1) (t + .3) (t + 1)) + t
```

```
t + 1.6 (-1 + t) (0.3 + t) (1 + t)
```

```
Plot[exp2b[t], {t, -1, 1}, ImageSize → 200,
 PlotRange → {{-1, 1}, {-1, 1}}, AspectRatio → 1]
```

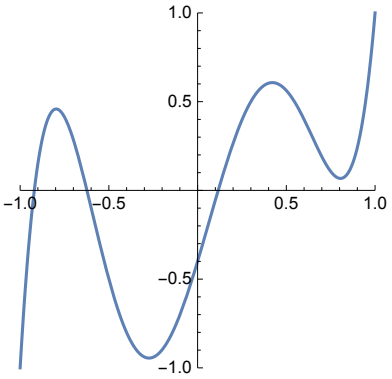



```
InterpolatingPolynomial[{{-1, -1}, {0, 1}, {1, -1}}, t]
```

```
-1 + (2 - 2 t) (1 + t)
```

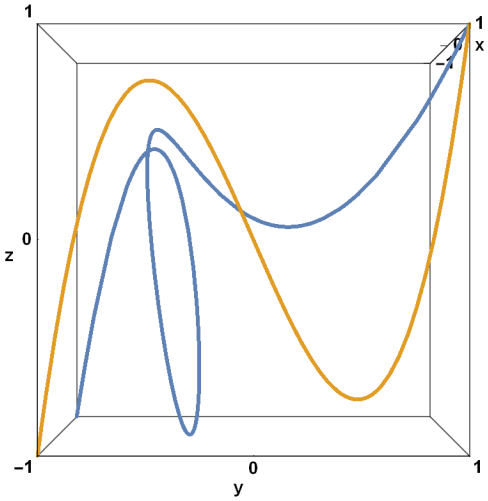
```
exp2c[t_] := InterpolatingPolynomial[
  {{-1, -1}, {-.5, -.5}, {0, -.4}, {.3, .5}, {.6, .4}, {1, 1}}, t]
```

```
Plot[exp2c[t], {t, -1, 1}, ImageSize → 200,
 PlotRange → {{-1, 1}, {-1, 1}}, AspectRatio → 1]
```

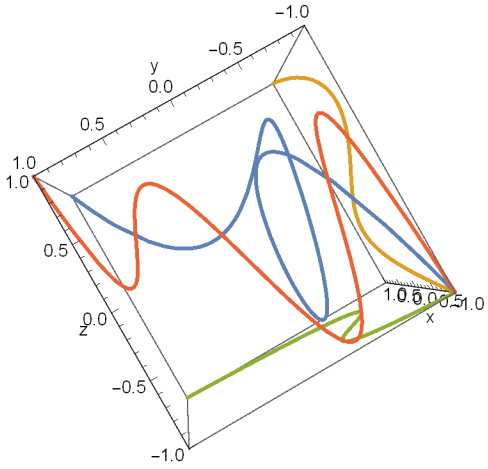



```
axst[t_] := Style[t, 14, Bold]
```

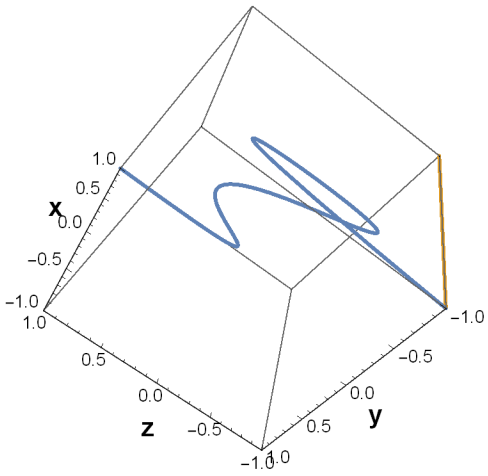
```
ParametricPlot3D[{{exp2a[t], exp2b[t], exp2c[t]},
  {1, t, exp2a[t]}}, {t, -1, 1},
 ImageSize → 250, PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}},
 AxesLabel → {"x", "y", "z"}, LabelStyle → Directive[Black, Bold],
 ViewPoint → {5, 0, 0}, Ticks → {{-1, 0, 1}, {-1, 0, 1}, {-1, 0, 1}}]
```




```
ParametricPlot3D[{{exp2a[t], exp2b[t], exp2c[t]},
  {exp2a[t], -1, t}, {t, exp2b[t], -1}, {-1, t, exp2c[t]}}, {t, -1, 1},
 ImageSize → 250, PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}}, AxesLabel → {"x", "y", "z"}]
```



```
ParametricPlot3D[{{exp2a[t], exp2b[t], exp2c[t]},
  {exp2a[t], -1, -1}},
 {t, -1, 1}, ImageSize → 250, PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}},
 AxesLabel → {axst["x"], axst["y"], axst["z"]}
]
```



```
Manipulate[
 ParametricPlot3D[{{exp2a[t], exp2b[t], exp2c[t]}},
  {t, -1, 1}, ImageSize → 250, PlotRange → {{-1, 1}, {-1, 1}, {-1, 1}},
  AxesLabel → {"x", "y", "z"}, ViewPoint → {a, b, c}],
 {{a, Pi}, 1, 50}, {{b, Pi/2}, 1, 50}, {{c, 2}, 1, 50}]
```